

On Wess-Zumino terms of Brane-Antibrane systems

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Abstract

We calculate the disk level S-matrix element of one closed string RR field, two open string tachyons and one gauge field in type II superstring theory. An expansion for the S-matrix element has been found that its four leading order terms are reproduced exactly by the symmetric trace tachyon DBI and the Wess-Zumino actions of D-brane-anti-D-brane systems. Using this consistency, we have also found the first higher derivative correction to the some of the WZ terms.

1 Introduction

Study of unstable objects in string theory might shed new light in understanding properties of string theory in time-dependent backgrounds [1, 2, 3, 4, 5, 6]. Generally speaking, source of instability in these processes is appearance of some tachyonic modes in the spectrum of these objects. It then makes sense to study them in a field theory which includes those modes. In this regard, it has been shown by A. Sen that an effective action of Born-Infeld type proposed in [7, 8, 9, 10] can capture many properties of the decay of non-BPS D_p -branes in string theory [2, 3].

Recently, unstable objects have been used to study spontaneous chiral symmetry breaking in holographic model of QCD [11, 12, 13]. In these studies, flavor branes introduced by placing a set of parallel branes and antibranes on a background dual to a confining color theory [14]. Detailed study of brane-antibrane system reveals when brane separation is smaller than the string length scale, spectrum of this system has two tachyonic modes [15]. The effective action should then include these modes because they are the most important ones which rule the dynamics of the system.

The effective action of a $D_p\bar{D}_p$ -brane in Type IIA(B) theory should be given by some extension of the DBI action and the WZ terms which include the tachyon fields. The DBI part may be given by the projection of the effective action of two non-BPS D_p -branes in Type IIB(A) theory with $(-1)^{F_L}$ [16]. We are interested in this paper in the appearance of tachyon, gauge field and the RR field in these actions. These fields appear in the DBI part as the following [17]:

$$S_{DBI} = - \int d^{p+1} \sigma \text{Tr} \left(V(\mathcal{T}) \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + 2\pi\alpha' D_a \mathcal{T} D_b \mathcal{T})} \right), \quad (1)$$

The trace in the above action should be completely symmetric between all matrices of the form F_{ab} , $D_a \mathcal{T}$, and individual \mathcal{T} of the tachyon potential. These matrices are

$$F_{ab} = \begin{pmatrix} F_{ab}^{(1)} & 0 \\ 0 & F_{ab}^{(2)} \end{pmatrix}, \quad D_a \mathcal{T} = \begin{pmatrix} 0 & D_a T \\ (D_a T)^* & 0 \end{pmatrix}, \quad \mathcal{T} = \begin{pmatrix} 0 & T \\ T^* & 0 \end{pmatrix} \quad (2)$$

where $F_{ab}^{(i)} = \partial_a A_b^{(i)} - \partial_b A_a^{(i)}$ and $D_a T = \partial_a T - i(A_a^{(1)} - A_a^{(2)})T$. If one uses ordinary trace, instead, the above action reduces to the action proposed by A.Sen [18] after making the kinetic term symmetric and performing the trace. This latter action is not consistent with S-matrix calculation. The tachyon potential which is consistent with S-matrix element calculations has the following expansion:

$$V(|T|) = 1 + \pi\alpha' m^2 |T|^2 + \frac{1}{2}(\pi\alpha' m^2 |T|^2)^2 + \dots$$

where T_p is the p-brane tension, m^2 is the mass squared of tachyon, *i.e.*, $m^2 = -1/(2\alpha')$. The above expansion is consistent with the potential $V(|T|) = e^{\pi\alpha'm^2|T|^2}$ which is the tachyon potential of BSFT [19].

The terms of the above action which has contribution to the S-matrix element of one gauge field and two tachyons in which we are interested in this paper are the following [17]:

$$\begin{aligned}\mathcal{L}_{DBI} = & -T_p(2\pi\alpha') \left(m^2|T|^2 + DT \cdot (DT)^* - \frac{\pi\alpha'}{2} (F^{(1)} \cdot F^{(1)} + F^{(2)} \cdot F^{(2)}) \right) + T_p(\pi\alpha')^3 \\ & \times \left(\frac{2}{3} DT \cdot (DT)^* (F^{(1)} \cdot F^{(1)} + F^{(1)} \cdot F^{(2)} + F^{(2)} \cdot F^{(2)}) \right. \\ & + \frac{2m^2}{3} |\tau|^2 (F^{(1)} \cdot F^{(1)} + F^{(1)} \cdot F^{(2)} + F^{(2)} \cdot F^{(2)}) \\ & \left. - \frac{4}{3} ((D^\mu T)^* D_\beta T + D^\mu T (D_\beta T)^*) (F^{(1)\mu\alpha} F_{\alpha\beta}^{(1)} + F^{(1)\mu\alpha} F_{\alpha\beta}^{(2)} + F^{(2)\mu\alpha} F_{\alpha\beta}^{(2)}) \right)\end{aligned}\quad (3)$$

Note that if one uses the on-shell value for the tachyon mass, *i.e.*, $m^2 = -1/(2\alpha')$, the above terms would not be ordered in terms of power of α' .

The WZ term describing the coupling of RR field to gauge field of brane-anti-brane is given by [20, 21]

$$S = \mu_p \int_{\Sigma_{(p+1)}} C \wedge (e^{i2\pi\alpha' F^{(1)}} - e^{i2\pi\alpha' F^{(2)}}), \quad (4)$$

where $\Sigma_{(p+1)}$ is the world volume and μ_p is the RR charge of the branes. In above equation, C is a formal sum of the RR potentials $C = \sum_n (-i)^{\frac{p-m+1}{2}} C_m$. Note that the factors of i disappear in each term of (4). The inclusion of the tachyon fields into this action has been proposed in [22, 23, 24] using the superconnection of noncommutative geometry [25, 26, 27]

$$S_{WZ} = \mu_p \int_{\Sigma_{(p+1)}} C \wedge \text{STr} e^{i2\pi\alpha' \mathcal{F}} \quad (5)$$

where the curvature of the superconnection is defined as:

$$\mathcal{F} = d\mathcal{A} - i\mathcal{A} \wedge \mathcal{A} \quad (6)$$

the superconnection is

$$i\mathcal{A} = \begin{pmatrix} iA^{(1)} & \beta T^* \\ \beta T & iA^{(2)} \end{pmatrix},$$

where β is a normalization constant with dimension $1/\sqrt{\alpha'}$ which we shall find it later, and a “supertrace” is defined by

$$\text{STr} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \text{Tr} A - \text{Tr} D.$$

Using the multiplication rule of the supermatrices [23]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} AA' + (-)^{c'} BC' & AB' + (-)^{d'} BD' \\ DC' + (-)^{a'} CA' & DD' + (-)^{b'} CB' \end{pmatrix} \quad (7)$$

where x' is 0 if X is an even form or 1 if X is an odd form, one finds that the curvature is

$$i\mathcal{F} = \begin{pmatrix} iF^{(1)} - \beta^2|T|^2 & \beta(DT)^* \\ \beta DT & iF^{(2)} - \beta^2|T|^2 \end{pmatrix},$$

where $F^{(i)} = \frac{1}{2}F_{ab}^{(i)}dx^a \wedge dx^b$ and $DT = [\partial_a T - i(A_a^{(1)} - A_a^{(2)})T]dx^a$. The WZ action (5) has the following terms:

$$\begin{aligned} C \wedge \text{STr } i\mathcal{F} &= C_{p-1} \wedge (F^{(1)} - F^{(2)}) \\ C \wedge \text{STr } i\mathcal{F} \wedge i\mathcal{F} &= C_{p-3} \wedge \{F^{(1)} \wedge F^{(1)} - F^{(2)} \wedge F^{(2)}\} \\ &\quad + C_{p-1} \wedge \{-2\beta^2|T|^2(F^{(1)} - F^{(2)}) + 2i\beta^2 DT \wedge (DT)^*\} \\ C \wedge \text{STr } i\mathcal{F} \wedge i\mathcal{F} \wedge i\mathcal{F} &= C_{p-5} \wedge \{F^{(1)} \wedge F^{(1)} \wedge F^{(1)} - F^{(2)} \wedge F^{(2)} \wedge F^{(2)}\} \\ &\quad + C_{p-3} \wedge \{-3\beta^2|T|^2(F^{(1)} \wedge F^{(1)} - F^{(2)} \wedge F^{(2)}) \\ &\quad \quad + 3i\beta^2(F^{(1)} + F^{(2)}) \wedge DT \wedge (DT)^*\} \\ &\quad + C_{p-1} \wedge \{3\beta^4|T|^4 \wedge (F^{(1)} - F^{(2)}) - 6i\beta^4|T|^2 DT \wedge (DT)^*\} \end{aligned} \quad (8)$$

The appearance of $C_{p-1} \wedge dT \wedge dT^*$ has been checked in [22] by studying the disk level S-matrix element of one RR field and two tachyons. In the present paper we will check, among other things, the appearance of $C_{p-1} \wedge DT \wedge (DT)^*$ and $C_{p-1} \wedge |T|^2(F^{(1)} - F^{(2)})$ terms and fix their coefficients using the S-matrix element of one RR field, two tachyons and one gauge field. The coupling of one RR field, two tachyons and one gauge field in the above terms can be combined into the following form:

$$\begin{aligned} &\mu_p(2\pi\alpha')^2(-\beta^2) \int_{\Sigma_{(p+1)}} C_{(p-1)} \wedge \{d(A^{(1)} - A^{(2)})TT^* - (A^{(1)} - A^{(2)})d(TT^*)\} \\ &= \mu_p(2\pi\alpha')^2(-\beta^2) \int_{\Sigma_{p+1}} H_{(p)} \wedge (A^{(1)} - A^{(2)})TT^* \end{aligned} \quad (9)$$

This combination actually appears naturally in the S-matrix element in the string theory side.

An outline of the rest of paper is as follows. In the next section, we review the calculate of the S-matrix element of one RR and two tachyons [22]. The low energy expansion of this amplitude produces one massless pole and infinite number of contact terms. The massless pole is the one reproduce by the above field theories, and its first contact term by

$C_{p-1} \wedge dT \wedge dT^*$. This fixes the normalization of tachyon in WZ action with respect to the tachyon in the DBI part. In section 3, we calculate the S-matrix element of one RR, two tachyons and one gauge field. We find an expansion for the amplitude whose leading order terms are fully consistent with the above field theories.

2 The $T - T - C$ amplitude

The three-point amplitude between one RR field and two tachyons has been studied in [22] where a non-zero coupling $C \wedge dT \wedge dT^*$ has been found. To fix the coefficient of this term we reexamine this amplitude in this section. The tachyon vertex operator corresponds to the real components of the complex tachyon, *i.e.*,

$$T = \frac{1}{\sqrt{2}}(T_1 + iT_2) \quad (10)$$

The three point amplitude between one RR and two tachyons is given by the following correlation functions:

$$\mathcal{A}^{T,T,RR} \sim \int dx dy d^2 w \langle V_T^{(0)}(x) V_T^{(-1)}(y) V_{RR}^{(-1)}(w, \bar{w}) \rangle \quad (11)$$

where the vertex operators are

$$\begin{aligned} V_T^{(0)}(x) &= 2ik \cdot \psi(x) e^{2ik \cdot X(x)} \\ V_T^{(-1)}(y) &= e^{-\phi(y)} e^{2ik' \cdot X(y)} \\ V_{RR}^{(-1)}(w, \bar{w}) &= (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} e^{-\phi(w)/2} S_\alpha(w) e^{ip \cdot X(w)} e^{-\phi(\bar{w})/2} S_\beta(\bar{w}) e^{ip \cdot D \cdot X(\bar{w})} \end{aligned} \quad (12)$$

The tachyon vertex operator corresponds to either T_1 or T_2 . The projector in the RR vertex operator is $P_- = \frac{1}{2}(1 - \gamma^{11})$ and

$$\mathbb{H}_{(n)} = \frac{a_n}{n!} H_{\mu_1 \dots \mu_n} \gamma^{\mu_1} \dots \gamma^{\mu_n} ,$$

where $n = 2, 4$ for type IIA and $n = 1, 3, 5$ for type IIB. $a_n = i$ for IIA and $a_n = 1$ for IIB theory. The spinorial indices are raised with the charge conjugation matrix, eg $(P_- \mathbb{H}_{(n)})^{\alpha\beta} = C^{\alpha\delta} (P_- \mathbb{H}_{(n)})_{\delta}{}^{\beta}$ (further conventions and notations for spinors can be found in appendix B of [28]). The RR bosons are massless so $p^2 = 0$ and for the tachyons $k^2 = k'^2 = 1/4$. In the string theory calculation we always set $\alpha' = 2$. The world-sheet fields has been extended to the entire complex plane. That is, we have replaced

$$\tilde{X}^\mu(\bar{w}) \rightarrow D_\nu^\mu X^\nu(\bar{w}) , \quad \tilde{\psi}^\mu(\bar{w}) \rightarrow D_\nu^\mu \psi^\nu(\bar{w}) , \quad \tilde{\phi}(\bar{w}) \rightarrow \phi(\bar{w}) , \quad \text{and} \quad \tilde{S}_\alpha(\bar{w}) \rightarrow M_\alpha{}^\beta S_\beta(\bar{w}) ,$$

where

$$D = \begin{pmatrix} 1_{p+1} & 0 \\ 0 & -1_{9-p} \end{pmatrix}, \quad \text{and} \quad M_p = \begin{cases} \frac{\pm i}{(p+1)!} \gamma^{a_0} \gamma^{a_1} \dots \gamma^{a_p} \epsilon_{a_0 \dots a_p} & \text{for } p \text{ even} \\ \frac{\pm 1}{(p+1)!} \gamma^{a_0} \gamma^{a_1} \dots \gamma^{a_p} \gamma_{11} \epsilon_{a_0 \dots a_p} & \text{for } p \text{ odd} \end{cases}$$

Using these replacements, one finds the standard propagators for the world-sheet fields X^μ, ϕ , *i.e.*,

$$\begin{aligned} \langle X^\mu(z) X^\nu(w) \rangle &= -\eta^{\mu\nu} \log(z-w), \\ \langle \phi(z) \phi(w) \rangle &= -\log(z-w). \end{aligned} \quad (13)$$

One also needs the correlation function between two spin operators and one ψ . The correlation function involving an arbitrary number of ψ 's and two S 's is obtained using the following Wick-like rule [29]:

$$\begin{aligned} \langle \psi^{\mu_1}(y_1) \dots \psi^{\mu_n}(y_n) S_\alpha(z) S_\beta(\bar{z}) \rangle &= \frac{1}{2^{n/2}} \frac{(z-\bar{z})^{n/2-5/4}}{|y_1-z| \dots |y_n-z|} \left[(\Gamma^{\mu_n \dots \mu_1} C^{-1})_{\alpha\beta} \right. \\ &\quad + \langle \psi^{\mu_1}(y_1) \psi^{\mu_2}(y_2) \rangle (\Gamma^{\mu_n \dots \mu_3} C^{-1})_{\alpha\beta} \pm perms \\ &\quad + \langle \psi^{\mu_1}(y_1) \psi^{\mu_2}(y_2) \rangle \langle \psi^{\mu_3}(y_3) \psi^{\mu_4}(y_4) \rangle (\Gamma^{\mu_n \dots \mu_5} C^{-1})_{\alpha\beta} \\ &\quad \left. \pm perms + \dots \right] \end{aligned} \quad (14)$$

where dots means sum over all possible contractions. In above equation, $\Gamma^{\mu_n \dots \mu_1}$ is the totally antisymmetric combination of the gamma matrices and the Wick-like contraction, for real y_i , is given by

$$\langle \psi^\mu(y_1) \psi^\nu(y_2) \rangle = 2\eta^{\mu\nu} \frac{Re[(y_1-z)(y_2-\bar{z})]}{(y_1-y_2)(z-\bar{z})}$$

The number of ψ in the correlators (11) is one. Using the above formula for one ψ and performing the other correlators using (13), one finds that the integrand is invariant under $SL(2, R)$ transformation. Gauge fixing this symmetry by fixing the position of vertex operators at $(x, y, w, \bar{w}) = (x, -x, i, -i)$, one finds [22]

$$\begin{aligned} \mathcal{A}^{T,T,RR} &\sim \int_{-\infty}^{\infty} dx \left(\frac{(1+x^2)^2}{16x^2} \right)^{\frac{1}{2}+u} \frac{2}{1+x^2} \text{Tr} (P_- \not{H}_{(n)} M_p \gamma^a) k_a, \\ &= \left(\frac{i\mu_p}{4} \right) 2\pi \frac{\Gamma[-2u]}{\Gamma[\frac{1}{2}-u]^2} \text{Tr} (P_- \not{H}_{(n)} M_p \gamma^a) k_a. \end{aligned} \quad (15)$$

where $u = -(k+k')^2$ and conservation of momentum along the world volume of brane, *i.e.*, $k^a + k'^a + p^a = 0$, has been used. We have also normalized the amplitude by $i\mu_p/4$. The

trace is zero for $p \neq n$, and for $n = p$ it is

$$\text{Tr} \left(\mathbb{H}_{(n)} M_p \gamma^a \right) = \pm \frac{32}{p!} H_{a_0 \dots a_{p-1}} \epsilon^{a_0 \dots a_{p-1} a} .$$

We are going to compare string theory S-matrix elements with field theory S-matrix element including their coefficients, however, we are not interested in fixing the overall sign of the amplitudes. Hence, in above and in the rest of equations in this paper, we have payed no attention to the sign of equations. The trace in (15) containing the factor of γ^{11} ensures the following results also hold for $p > 3$ with $H_{(n)} \equiv *H_{(10-n)}$ for $n \geq 5$.

If one replaces k_a in (15) with $-k'_a - p_a$ and uses using the conservation of momentum, one will find that the p_a term vanishes using the totally antisymmetric property of $\epsilon^{a_0 \dots a_{p-1} a}$. Hence the amplitude (15) is antisymmetric under interchanging $1 \leftrightarrow 2$. This indicates that the three point amplitude between one RR and two T_1 or two T_2 is zero.



Figure 1 : The Feynman diagrams corresponding to the amplitudes (16) and (19).

The effective Lagrangian of massless and tachyonic particles, (3) and (8), produces the following massless pole:

$$\mathcal{A} = V_a(C_{p-1}, A) G_{ab}(A) V_b(A, T_1, T_2) \quad (16)$$

where the gauge field in the off-shell line is $A^{(1)}$ and $A^{(2)}$. The propagator and vertexes are

$$\begin{aligned} G_{ab}(A) &= \frac{i\delta_{ab}}{(2\pi\alpha')^2 T_p (-p_a^2)} \\ V_b(A^{(1)}, T_1, T_2) &= iT_p(2\pi\alpha')(k_b - k'_b) \\ V_b(A^{(2)}, T_1, T_2) &= -iT_p(2\pi\alpha')(k_b - k'_b) \\ V_a(C_{p-1}, A^{(1)}) &= i\mu_p(2\pi\alpha') \frac{1}{p!} \epsilon_{a_0 \dots a_{p-1} a} H^{a_0 \dots a_{p-1}} \\ V_a(C_{p-1}, A^{(2)}) &= -i\mu_p(2\pi\alpha') \frac{1}{p!} \epsilon_{a_0 \dots a_{p-1} a} H^{a_0 \dots a_{p-1}} \end{aligned} \quad (17)$$

Replacing them in (16), one finds

$$\mathcal{A} = 4i\mu_p \frac{1}{p!u} \epsilon_{a_0 \dots a_{p-1}a} H^{a_0 \dots a_{p-1}} k^a \quad (18)$$

The field theory (8) has also the following contact term:

$$\mathcal{A}_c = i\mu_p (2\pi\alpha')^2 \beta^2 \frac{1}{p!} \epsilon_{a_0 \dots a_{p-1}a} H^{a_0 \dots a_{p-1}} k^a \quad (19)$$

The massless pole of field theory (18) can be obtained from the S-matrix element (15) by expanding it at low energy ($u = -p_a^2 \rightarrow 0$). The prefactor of (15) has the expansion

$$2\pi \frac{\Gamma[-2u]}{\Gamma[\frac{1}{2}-u]^2} = \frac{-1}{u} + 4\ln(2) + \left(\frac{\pi^2}{6} - 8\ln(2)^2 \right) u + O(u^2) . \quad (20)$$

The first term is exactly the field theory massless pole (18). The second term should be the contact term in (19). This fixes the normalization constant β to be

$$\beta = \frac{1}{\pi} \sqrt{\frac{2\ln(2)}{\alpha'}} \quad (21)$$

Note that in the string theory calculations we have set $\alpha' = 2$, so to compare the string theory amplitudes with the corresponding amplitudes in field theory one has to set $\alpha' = 2$ in the field theory side too.

Since the expansion (20) is in terms of the powers of p_a^2 , the other terms in (20) correspond to the higher derivative corrections of the WZ action. For example, it is easy to check that the following higher derivative term reproduces the third term in (20):

$$i(\alpha')^2 \mu_p \left(\frac{\pi^2}{6} - 8\ln(2)^2 \right) C_{p-1} \wedge D^a D_a (DT \wedge DT^*) \quad (22)$$

On the other hand, one may write $u = -p_a^2 = -1/2 - 2k \cdot k'$ using the on-shell condition $k^2 = k'^2 = 1/4$. Then one may conclude that because of the $-1/2$ term, the last term in (20) does not correspond to the higher derivative of the tachyons. As we mentioned, it is obvious that the $O(u)$ terms in the expansion (20) correspond to the higher derivative of the tachyons, however, for other S-matrix elements, *e.g.*, the S-matrix element of four tachyons, it is hard to prove it. It is speculated though that the non-leading terms of the expansion of any S-matrix element of tachyons correspond to the higher derivative of the tachyon field [30].

It is interesting to compare the above higher derivative term with the higher derivative term of one RR and two gauge fields. The string theory S-matrix element of one RR and two gauge fields is given by [31, 32]

$$\mathcal{A} \sim 2 \frac{\Gamma[-2u]}{\Gamma[1-u]^2} K \quad (23)$$

where K is the kinematic factor. Expansion of the prefactor at low energy is

$$2 \frac{\Gamma[-2u]}{\Gamma[1-u]^2} = \frac{-1}{u} - \left(\frac{\pi^2}{6} \right) u + O(u^2) . \quad (24)$$

In this case actually there is no massless pole at $u = 0$ as the kinematic factor provides a compensating factor of u . The amplitude has the following expansion:

$$\mathcal{A} = i \frac{(4\pi)^2 \mu_p}{4(p-3)!} f^{a_0 a_1} f'^{a_2 a_3} \varepsilon^{a_4 \dots a_p} \epsilon_{a_0 \dots a_p} \left(1 + \left(\frac{\pi^2}{6} \right) u^2 + O(u^3) \right) \delta_{p,n+2} \quad (25)$$

where $f_{ab} = i(k_a \xi_b - k_b \xi_a)$, $f'_{ab} = i(k'_a \xi'_b - k'_b \xi'_a)$ and ε is the polarization of the RR potential. The first term is reproduced by the couplings in the ZW terms (8), and the second term by the following higher derivative term

$$\frac{(\pi\alpha')^2 \mu_p}{6} \frac{\mu_p}{2!} (2\pi\alpha')^2 C_{p-3} \wedge \left(\partial^a \partial^b F^{(1)} \wedge \partial_a \partial_b F^{(1)} - \partial^a \partial^b F^{(2)} \wedge \partial_a \partial_b F^{(2)} \right) \quad (26)$$

While the leading higher derivative term in (22) has two extra derivatives with respect to the corresponding coupling in the WZ terms, in above coupling there are four extra derivatives with respect to the corresponding coupling in (8).

3 The $T - T - A - C$ amplitude

The S-matrix element of one RR field, two tachyons and one gauge field is given by the following correlation function:

$$\mathcal{A}^{ATT C} \sim \int dx_1 dx_2 dx_3 dz d\bar{z} \langle V_A^{(-1)}(x_1) V_T^{(0)}(x_2) V_T^{(0)}(x_3) V_{RR}^{(-1)}(z, \bar{z}) \rangle \quad (27)$$

where we have chosen the vertex operators according to the rule that the total superghost number must be -2 . The tachyon and RR vertex operators are given in (12) and the gauge field vertex operator in (-1)-picture is

$$V_A^{(-1)} = \xi \cdot \psi(x_1) e^{-\phi(x_1)} e^{2ik_1 \cdot X(x_1)} \quad (28)$$

where ξ_a is the gauge field polarization. Introducing $x_4 \equiv z = x + iy$ and $x_5 \equiv \bar{z} = x - iy$, the scattering amplitude reduces to the following correlators:

$$\begin{aligned}\mathcal{A}^{ATT^*C} &\sim \int dx_1 \cdots dx_5 (P_- \not{H}_{(n)} M_p)^{\alpha\beta} \xi_a <: e^{-1/2\phi(x_4)} : e^{-1/2\phi(x_5)} : e^{-\phi(x_1)} :> \\ &\times <: e^{2ik_1 \cdot X(x_1)} : e^{2ik_2 \cdot X(x_2)} : e^{2ik_3 \cdot X(x_3)} : e^{ip \cdot X(x_4)} : e^{ip \cdot D \cdot X(x_5)} :> \\ &\times <: S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_1) : 2k_2 \cdot \psi(x_2) : 2k_3 \cdot \psi(x_3) :>\end{aligned}$$

The correlators in the first and the second lines can be calculated using the propagators in (13), and the correlator in the last line can read from (14). The result is

$$\begin{aligned}\mathcal{A}^{ATTC} &\sim \sqrt{2} \int dx_1 \cdots dx_5 k_{2b} k_{3c} \xi_a (x_{34} x_{35})^{-1/2+2k_3 \cdot p} (x_{14} x_{15})^{-1+2k_1 \cdot p} (x_{24} x_{25})^{-1/2+2k_2 \cdot p} \\ &\times x_{45}^{p \cdot D \cdot p} x_{23}^{4k_2 \cdot k_3} x_{12}^{4k_1 \cdot k_2} x_{13}^{4k_1 \cdot k_3} (P_- \not{H}_{(n)} M_p)^{\alpha\beta} \left\{ (\Gamma^{cba} C^{-1})_{\alpha\beta} + 2\eta^{ab} \frac{Re[x_{14} x_{25}]}{x_{12} x_{45}} (\Gamma^c C^{-1})_{\alpha\beta} \right. \\ &\left. - 2\eta^{ac} \frac{Re[x_{14} x_{35}]}{x_{13} x_{45}} (\Gamma^b C^{-1})_{\alpha\beta} + 2\eta^{bc} \frac{Re[x_{24} x_{35}]}{x_{23} x_{45}} (\Gamma^a C^{-1})_{\alpha\beta} \right\}\end{aligned}$$

where $x_{ij} \equiv x_i - x_j$. One can show that the integrand is invariant under $SL(2, \mathbb{R})$ transformation. Gauge fix this symmetry by fixing

$$x_1 = 0, \quad x_2 = 1, \quad x_3 \rightarrow \infty, \quad dx_1 dx_2 dx_3 \rightarrow x_3^2$$

One finds

$$\begin{aligned}\mathcal{A}^{ATTC} &\sim \sqrt{2} \int dx_4 dx_5 k_{2b} k_{3c} \xi_a x_{45}^{-2(t+s+u)-1} |x_4|^{2t+2s-1} |1-x_4|^{2u+2t-1/2} \\ &\times \left\{ (\Gamma^{cba} C^{-1})_{\alpha\beta} - 2\eta^{ab} \frac{-x + x_4 x_5}{x_{45}} (\Gamma^c C^{-1})_{\alpha\beta} - 2\eta^{ac} \frac{x}{x_{45}} (\Gamma^b C^{-1})_{\alpha\beta} \right. \\ &\left. + 2\eta^{bc} \frac{x-1}{x_{45}} (\Gamma^a C^{-1})_{\alpha\beta} \right\} (P_- \not{H}_{(n)} M_p)^{\alpha\beta}\end{aligned}$$

where we have also introduced the Mandelstam variables

$$s = -(k_1 + k_3)^2, \quad t = -(k_1 + k_2)^2, \quad u = -(k_2 + k_3)^2$$

and used the conservation of momentum along the brane, *i.e.*, $k_1^a + k_2^a + k_3^a + p^a = 0$.

Using the fact that M_p , $\not{H}_{(n)}$, and Γ^{cba} are totally antisymmetric combinations of the Gamma matrices, one realizes that the first term is non-zero only for $p = n + 2$, and the last three terms are non-zero only for $p = n$. The integral in the above equation can be written in terms of the Gamma functions, using the following identity [33]:

$$\int d^2 z |1-z|^a |z|^b (z-\bar{z})^c (z+\bar{z})^d = (2i)^c 2^d \pi \frac{\Gamma(1+d+\frac{b+c}{2}) \Gamma(1+\frac{a+c}{2}) \Gamma(-1-\frac{a+b+c}{2}) \Gamma(\frac{1+c}{2})}{\Gamma(-\frac{a}{2}) \Gamma(-\frac{b}{2}) \Gamma(2+c+d+\frac{a+b}{2})}$$

for $d = 0, 1$ and arbitrary a, b, c . The region of integration is the upper half complex plane, as in our case. Using this integral, one finds

$$\begin{aligned} \mathcal{A}^{ATTC} &= \frac{i\mu_p}{2\sqrt{2\pi}} \left[\text{Tr} \left((P_- \not{H}_{(n)} M_p) (k_3 \cdot \gamma) (k_2 \cdot \gamma) (\xi \cdot \gamma) \right) I \delta_{p,n+2} + \text{Tr} \left((P_- \not{H}_{(n)} M_p) \gamma^a \right) J \delta_{p,n} \right. \\ &\quad \left. \times \left\{ k_{2a}(t + 1/4)(2\xi \cdot k_3) + k_{3a}(s + 1/4)(2\xi \cdot k_2) - \xi_a(s + 1/4)(t + 1/4) \right\} \right] \end{aligned} \quad (29)$$

where we have normalized the amplitude by $i\mu_p/2\sqrt{2\pi}$. In above equation, I, J are :

$$\begin{aligned} I &= 2^{1/2} (2)^{-2(t+s+u)-1} \pi \frac{\Gamma(-u)\Gamma(-s+1/4)\Gamma(-t+1/4)\Gamma(-t-s-u)}{\Gamma(-u-t+1/4)\Gamma(-t-s+1/2)\Gamma(-s-u+1/4)} \\ J &= 2^{1/2} (2)^{-2(t+s+u+1)} \pi \frac{\Gamma(-u+1/2)\Gamma(-s-1/4)\Gamma(-t-1/4)\Gamma(-t-s-u-1/2)}{\Gamma(-u-t+1/4)\Gamma(-t-s+1/2)\Gamma(-s-u+1/4)} \end{aligned}$$

The traces are:

$$\begin{aligned} \text{Tr} \left(\not{H}_{(n)} M_p (k_3 \cdot \gamma) (k_2 \cdot \gamma) (\xi \cdot \gamma) \right) \delta_{p,n+2} &= \pm \frac{32}{n!} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_{p-3}} k_{3a_{p-2}} k_{2a_{p-1}} \xi_{a_p} \delta_{p,n+2} \\ \text{Tr} \left(\not{H}_{(n)} M_p \gamma^a \right) \delta_{p,n} &= \pm \frac{32}{n!} \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}} \delta_{p,n} \end{aligned} \quad (30)$$

Examining the poles of the Gamma functions, one realizes that for the case that $p = n + 2$, the amplitude has massless pole and infinite tower of massive poles. Whereas for $p = n$ case, there are tachyon, massless, and infinite tower of massive poles. Now the non trivial question is how to expand this amplitude such that its leading terms correspond to the effective actions (3) and (8), and its non leading terms correspond to the higher derivative terms? Let us study each case separately.

3.1 $p = n + 2$ case

For $p = n + 2$, the amplitude is antisymmetric under interchanging $2 \leftrightarrow 3$, hence the four-point function between one RR, one gauge field and two T_1 or two T_2 is zero. The electric part of the amplitude for one RR, one gauge field, one T_1 and one T_2 is given by

$$\mathcal{A}^{AT_1 T_2 C} = \pm \frac{8i\mu_p}{\sqrt{2\pi}(p-2)!} \left[\epsilon^{a_0 \dots a_p} H_{a_0 \dots a_{p-3}} k_{3a_{p-2}} k_{2a_{p-1}} \xi_{a_p} \right] I \quad (31)$$

Note that the amplitude satisfies the Ward identity, *i.e.*, the amplitude vanishes under replacement $\xi^a \rightarrow k_1^a$.



Figure 2 : The Feynman diagrams corresponding to the amplitudes (32) and (35).

The effective Lagrangian of the massless field and tachyon, (3) and (8), produces the following massless pole for $p = n + 2$:

$$\mathcal{A} = V_a(C_{p-3}, A^{(1)}, A^{(1)}) G_{ab}(A^{(1)}) V_b(A^{(1)}, T_1, T_2) \quad (32)$$

where

$$\begin{aligned} G_{ab}(A^{(1)}) &= \frac{i\delta_{ab}}{(2\pi\alpha')^2 T_p(u)} \\ V_b(A^{(1)}, T_1, T_2) &= T_p(2\pi\alpha')(k_2 - k_3)_b \\ V_a(C_{p-3}, A^{(1)}, A^{(1)}) &= \mu_p(2\pi\alpha')^2 \frac{1}{(p-2)!} \epsilon_{a_0 \dots a_{p-1} a} H^{a_0 \dots a_{p-3}} k_1^{a_{p-2}} \xi^{a_{p-1}} \end{aligned} \quad (33)$$

The amplitude (32) becomes

$$\mathcal{A} = \mu_p(2\pi\alpha') \frac{2i}{(p-2)!u} \epsilon_{a_0 \dots a_{p-1} a} H^{a_0 \dots a_{p-3}} k_2^{a_{p-2}} k_3^{a_{p-1}} \xi^a \quad (34)$$

The WZ action (8) has also the following contact term:

$$\mathcal{A}_c = \mu_p \beta^2 (2\pi\alpha')^3 \frac{i}{2!(p-2)!} \epsilon_{a_0 \dots a_{p-1} a} H^{a_0 \dots a_{p-3}} k_2^{a_{p-2}} k_3^{a_{p-1}} \xi^a \quad (35)$$

The massless pole (34) and the contact term (35) can be exactly reproduced by the string theory amplitude (29) if one expands it around

$$t \rightarrow -1/4, \quad s \rightarrow -1/4, \quad u \rightarrow 0 \quad (36)$$

In fact expansion of I around this point is

$$\begin{aligned} I &= \pi\sqrt{2\pi} \left(\frac{-1}{u} + 4\ln(2) + \right. \\ &\quad \left. + \left(\frac{\pi^2}{6} - 8\ln(2)^2 \right) u - \frac{\pi^2 (s+t+1/2)^2}{6u} + \dots \right) \end{aligned} \quad (37)$$

replacing it in (31), one finds that the first term is reproduced by (34) and the second term by (35).

Now what about the other terms of (37)? A natural extension of the higher derivative term (22) to C_{p-3} which is similar to the extension of $C_{p-1} \wedge (DT \wedge DT^*)$ to C_{p-3} in (8) is the following:

$$\frac{i}{2}(2\pi\alpha')(\alpha')^2\mu_p\left(\frac{\pi^2}{6}-8\ln(2)^2\right)C_{p-3}\wedge(F^{(1)}+F^{(2)})\wedge D^aD_a(DT\wedge DT^*) \quad (38)$$

This higher derivative term in field theory reproduces *exactly* the contact term in the second line of (37). The last term in (37), on the other hand, is given by the Feynman amplitude (32) where the vertex $V_a(C_{p-3}, A^{(1)}, A^{(1)})$ should be derived from the higher derivative term (26), that is

$$V_a(C_{p-3}, A^{(1)}, A^{(1)}) = \frac{(2\pi)^2}{6}\mu_p(2\pi\alpha')^2\frac{1}{(p-2)!}\epsilon_{a_0\cdots a_{p-1}a}H^{a_0\cdots a_{p-3}}k_1^{a_{p-2}}\xi^{a_{p-1}}(k_1\cdot p)^2 \quad (39)$$

Note that the vertex $V_b(A^{(1)}, T_1, T_2)$ has no higher derivative correction as it arises from the kinetic term of the tachyon. The tachyon pole of string amplitude indicates that the kinetic term has no higher derivative correction. Now replacing $V_b(A^{(1)}, T_1, T_2)$ and the above vertex in (32) one again finds exact agreement with the string theory amplitude. Hence the expansion of the string amplitude (31) around (36) produces an expansion whose leading order terms are reproduced by the Feynman amplitudes resulting from DBI and WZ actions and the next terms are related to the higher derivative corrections to the WZ action. In particular, its massless poles gives information about the higher derivative corrections of $C_{p-3} \wedge F \wedge F$ and its contact terms about $C_{p-3} \wedge F \wedge DT \wedge DT^*$.

3.2 $p = n$ case

Now we consider $n = p$ case. The string theory amplitude in this case is symmetric under interchanging $2 \leftrightarrow 3$. On the other hand, there is no Feynman amplitude in field theory corresponding to four-point function of one RR, one gauge field, one T_1 and one T_2 . Hence, for $p = n$ the string theory amplitude (29) is the S-matrix element of one RR, one gauge field and two T_1 or two T_2 . Its electric part is,

$$\begin{aligned} \mathcal{A}^{AT_1T_1C} &= \pm \frac{8i\mu_p}{\sqrt{2\pi p!}} \left[\left(\epsilon^{a_0\cdots a_{p-1}a} H_{a_0\cdots a_{p-1}} \right) J \right. \\ &\quad \times \left. \left\{ k_{2a}(t+1/4)(2\xi\cdot k_3) + k_{3a}(s+1/4)(2\xi\cdot k_2) - \xi_a(s+1/4)(t+1/4) \right\} \right] \quad (40) \end{aligned}$$

Note that the amplitude satisfies the Ward identity, *i.e.*, the amplitude vanishes under replacement $\xi^a \rightarrow k_1^a$.



Figure 3 : The Feynman diagrams corresponding to the amplitudes in (41) and (44).

The effective field theory, (3) and (8), has the following massless pole for $p = n$:

$$\mathcal{A} = V_a(C_{p-1}, A) G_{ab}(A) V_b(A, T_1, T_1, A^{(1)}) \quad (41)$$

where A should be $A^{(1)}$ and $A^{(2)}$. The propagator and vertexes $V_a(C_{p-1}, A)$ and $V_b(A, T_1, T_1, A^{(1)})$ are

$$\begin{aligned} G_{ab}(A) &= \frac{i\delta_{ab}}{(2\pi\alpha')^2 T_p (u + t + s + 1/2)} \\ V_a(C_{p-1}, A^{(1)}) &= i\mu_p (2\pi\alpha') \frac{1}{p!} \epsilon_{a_0 \dots a_{p-1} a} H^{a_0 \dots a_{p-1}} \\ V_a(C_{p-1}, A^{(2)}) &= -i\mu_p (2\pi\alpha') \frac{1}{p!} \epsilon_{a_0 \dots a_{p-1} a} H^{a_0 \dots a_{p-1}} \\ V_b(A^{(1)}, T_1, T_1, A^{(1)}) &= -2iT_p (2\pi\alpha') \xi_b \\ V_b(A^{(2)}, T_1, T_1, A^{(1)}) &= 2iT_p (2\pi\alpha') \xi_b \end{aligned} \quad (42)$$

The amplitude (41) then becomes

$$\mathcal{A} = \frac{4i\mu_p}{p!(u + s + t + 1/2)} \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}} \xi_a \quad (43)$$

The couplings in (9) has also the following contact term:

$$\mathcal{A}_c = i\mu_p \ln(2) \frac{16}{p!} \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}} \xi_a \quad (44)$$

Now in string theory side, expansion of $(s + 1/4)(t + 1/4)J$ in (40) around (36) is

$$\begin{aligned} (s + 1/4)(t + 1/4)J &= \frac{\sqrt{2\pi}}{2} \left(\frac{-1}{(t + s + u + 1/2)} + 4\ln(2) \right. \\ &\quad \left. + \left(\frac{\pi^2}{6} - 8\ln(2)^2 \right) (s + t + u + 1/2) - \frac{\pi^2}{3} \frac{(t + 1/4)(s + 1/4)}{(t + s + u + 1/2)} + \dots \right) \end{aligned} \quad (45)$$

Replacing it in (40), one finds that the first term above is reproduced by the field theory massless pole (43) and the second term by (44). To find the higher derivative term corresponding to the first term in the second line above, we note that the WZ terms in the third line of (8) indicates that the following higher derivative term should accompany the term in (22):

$$- (\alpha')^2 \mu_p \left(\frac{\pi^2}{6} - 8 \ln(2)^2 \right) C_{p-1} \wedge D^a D_a [(F^{(1)} - F^{(2)}) |T|^2] \quad (46)$$

Combining the above with the coupling of one RR, two tachyons and one gauge field of (22), one finds the following coupling:

$$- (\alpha')^2 \mu_p \left(\frac{\pi^2}{6} - 8 \ln(2)^2 \right) H_p \wedge \partial^a \partial_a [(A^{(1)} - A^{(2)}) T T^*] \quad (47)$$

which reproduce exactly the first term in the second line of (45).

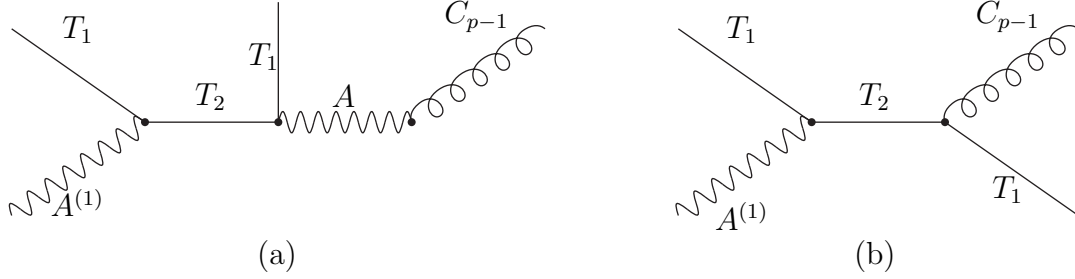


Figure 4 : The Feynman diagrams corresponding to the amplitudes (48).

The effective field theory has also the following poles for $p = n$:

$$\begin{aligned} \mathcal{A} = & V_a(C_{p-1}, A) G_{ab}(A) V_b(A, T_1, T_2) G(T_2) V(T_2, T_1, A^{(1)}) \\ & + V(C_{p-1}, T_1, T_2) G(T_2) V(T_2, T_1, A^{(1)}) \end{aligned} \quad (48)$$

where the off-shell gauge field A should be $A^{(1)}$ and $A^{(2)}$, and

$$\begin{aligned} V(C_{p-1}, T_1, T_2) &= \beta^2 \mu_p (2\pi\alpha') \frac{1}{p!} \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}} k_{2a} \\ V(T_2, T_1, A^{(1)}) &= T_p (2\pi\alpha') (k_3 - k) \cdot \xi \\ V_b(A, T_1, T_2) &= T_p (2\pi\alpha') (k_{2a} + k_a) \\ G(T_2) &= \frac{i}{(2\pi\alpha') T_p (s + 1/4)} \end{aligned} \quad (49)$$

where k_a is momentum of the off-shell tachyon. Replacing them in (48), one finds

$$4\mu_p \left(\frac{-1}{(s+1/4)(t+s+u+1/2)} + \frac{4\ln(2)}{(s+1/4)} \right) \frac{1}{p!} \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}} k_{2a} (2k_3 \cdot \xi) \quad (50)$$

Similar result appear for the Feynmann diagram in which $k_2 \leftrightarrow k_3$. Now in string theory side, expansion of $(t+1/4)J$ around (36) is

$$(t+1/4)J = \frac{1}{2}\sqrt{2\pi} \left(\frac{-1}{(s+1/4)(t+s+u+1/2)} + \frac{4\ln(2)}{(s+1/4)} + \left(\frac{\pi^2}{6} - 8\ln(2)^2 \right) \frac{(s+t+u+1/2)}{(s+1/4)} - \frac{\pi^2}{3} \frac{(t+1/4)}{(t+s+u+1/2)} + \dots \right) \quad (51)$$

Replacing it in the string theory amplitude (40), one finds exact agreement between the first two terms and the field theory results (50). The first term in the second line should be reproduced by the following Feynman amplitude in field theory:

$$\mathcal{A} = V(C_{p-1}, T_1, T_2) G(T_2) V(T_2, T_1, A^{(1)}) \quad (52)$$

where the propagator and the vertex $V(T_2, T_1, A^{(1)})$ is the same as in (49), however, the vertex $V(C_{p-1}, T_1, T_2)$ should be derived from the higher derivative term (22), that is

$$V(C_{p-1}, T_1, T_2) = (\alpha')^2 \mu_p \left(\frac{\pi^2}{6} - 8\ln(2)^2 \right) \frac{1}{p!} \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}} k_{2a} (s+t+u+1/2)$$

Replacing them in (52), one finds exact agreement with the string theory result. Finally, the sum of last term in (51) and the corresponding term when $k_2 \leftrightarrow k_3$, and the last term in (45) is

$$- \frac{4i}{3} \pi^2 \mu_p \frac{\epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}}}{p!(s+t+u+1/2)} \left[k_{2a} (t+1/4) (2\xi \cdot k_3) - \frac{1}{2} \xi_a (s+1/4) (t+1/4) + (1 \leftrightarrow 2) \right]$$

To find the corresponding term in field theory, consider the following Feynman amplitude:

$$\mathcal{A} = V_a(C_{p-1}, A) G_{ab}(A) V_b(A, A^{(1)}, T_1, T_1) \quad (53)$$

where the vertex $V_a(C_{p-1}, A)$ is the same as the one appears in (42) and $V_b(A, A^{(1)}, T_1, T_2)$ is derived from the second line of (3), that is

$$\begin{aligned} V_b(A^{(1)}, A^{(1)}, T_1, T_1) &= \frac{4i}{3} T_p (\pi\alpha')^3 k_b [(s+1/4)(2k_2 \cdot \xi) + (t+1/4)(2k_3 \cdot \xi)] + 2iT_p (\pi\alpha')^3 \times \\ &\quad \times [k_{2b}(t+1/4)(2\xi \cdot k_3) + k_{3b}(s+1/4)(2\xi \cdot k_2) - \xi_b(s+1/4)(t+1/4)] \\ V_b(A^{(2)}, A^{(1)}, T_1, T_2) &= \frac{2i}{3} T_p (\pi\alpha')^3 k_b [(s+1/4)(2k_2 \cdot \xi) + (t+1/4)(2k_3 \cdot \xi)] + 2iT_p (\pi\alpha')^3 \times \\ &\quad \times [k_{2b}(t+1/4)(2\xi \cdot k_3) + k_{3b}(s+1/4)(2\xi \cdot k_2) - \xi_b(s+1/4)(t+1/4)] \end{aligned}$$

where k^a is the momentum of the off-shell gauge field. Replacing them in (53), one finds exact agreement with the string theory result. Note that the $F^{(1)} \cdot F^{(2)}$ term in the tachyon DBI action (3) is necessary for the above consistency. The above consistency indicates that the coupling $C_{p-1} \wedge F$ has no higher derivative correction.

Now let us speculate on the other terms of the string theory expansion. Since there is no higher derivative corrections to $C_{p-1} \wedge F$ and to the kinetic term, the expansion should not have the double poles other than the one appears in the leading term. For the simple tachyonic poles, since the kinetic term has no correction, the non-leading poles of tachyon should give information about the higher derivative corrections to $C_{p-1} \wedge DT \wedge DT^*$. For the simple massless poles, since there is no correction to $C_{p-1} \wedge F$, the non-leading massless poles should give information about the higher derivative corrections to the coupling of two tachyons and two gauge fields. Finally, the contact terms of the string theory amplitude gives information about the higher derivative correction to $C_{p-1} \wedge DT \wedge DT^*$ and to $C_{p-1} \wedge FTT^*$. It would be interesting to study these terms in details and find higher derivative corrections to tachyon DBI and to WZ actions.

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